INTEGRATED GPS AND GLONASS SYSTEMS

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Abstract

Satellite Navigation Systems have not only revolutionized navigation, but also geodetic positioning. At present the Global Positioning System and the Russian Global Navigation Satellite based Global Navigation Systems. The major limiting factor with the most popular GPS system performance is the number and geometry of the satellites in view at any particular point in space and time. in order to alleviate this restriction, it stands to reason that more observations be introduced to provide increased levels of positioning continuity and higher levels of data redundancy. GLONASS measurements are thus introduced to the mathematical model to increase the number of available satellites measurements at any particular point and increase the geometrical strength of those observations available.

This paper describes the design implementation issues of integration of GPS and GLONASS particularly the mathematical models of GPS and GLONASS signal structure, differences in co-ordinate and time reference.

Introduction

Application of GPS is equivalent to those of GPS and can be seen mostly in highly precise navigation of land, sea, air and low orbiting spacecrafts. With number of applications and the achievable accuracy, GPS has become an attractive tool for navigational and geodetic purposes. But not only GPS as a stand-alone system draws the interest of scientists around the world. The fact that there are two independent, but generally very similar satellite navigation systems also draws attention to the combined use of both systems. This combined use brings up a number of advantages. At first the available satellites is increased with respect to one single system. Besides that, the more satellite measurements are available, the earlier and more reliable a user can detect and isolate malfunctioning satellites. Thus, the combined use of GPS and GLONASS may aid in receiver Autonomous Integrity Monitoring (RAIM), providing better integrity of the position fix than a single system alone. Basically the differences of GLONASS to GPS in terms of time frame and coordinate frame are worked out and these differences are worked out in combined GPS/GLONASS application. Apart from this, numbers of hardware implementation issues are also addressed in order to realize the combined system.

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Time Systems

GPS and GLONASS both use their own time scales, which, in addition, are connected to different realization of UTC. Therefore, GPS time and GLONASS time cannot easily be transformed from one time scale into the other. In combined GPS/GLONASS data processing the differences between these time scales must be accounted for. Otherwise, systematic errors are introduced that will affect the combined positioning solution.

GLONASS Time

GLONASS system time is maintained by the GLONASS Central Synchronizer. It is closely coupled to UTC, but with a constant offset of three hours(corresponding to the offset of Moscow time to Greenwich time).therefore GLONASS system time also considers leap seconds. Further differences between GLONASS time and UTC arise from the keeping of the time scales by two different master clocks.
The GLONASS user is informed to UTC as maintained by the National Etalon of Time and Frequency in Moscow (UTC_{SU}). This information is obtained from UTC parameter τ_c In frame 5 of the GLONASS ephemeredes message.

UTC then can be computed from GLONASS time according to the relation

$$t_{UTC} = t_{GLONASS} + \tau_c - 3h$$

The accuracy of this computed t_{UTC} is specified to be less than $1\mu s$. With time and frequency uploads to the satellite twice a day, this stability provides an accuracy of satellite time synchronization to system time of about 15 ns. Accuracy of the uploaded corrections is specified to be less than 35ns.

GPS Time

GPS system time is maintained by GPS Master Control Station. Since it is a uniform time scale, it differs from UTC by the leap seconds introduced into the latter time scale. Currently this difference is 13 seconds. In addition to the leap seconds, further differences between GPS system time and UTC arise from the fact that GPS system time and UTC are kept by different master clocks. The additional differences are in the order of nanoseconds. In fact GPS operators usually keep GPS system time to within 100ns of UTC as maintained by the US-Naval Observatory (UTC_{USNO}) .
The GPS user is informed about the difference to UTC_{USNO} . This information is obtained from the UTC parameters in page 18 of sub frame 4 of the GPS ephemeris message. This set of parameters consists of the following values:

1. The time given by WN_{LSF} and DN is not the past and the present time is not in the interval [DN+3/4,DN+5/4].

$$t_{UTC} = (t_{GPS} - \Delta t_{UTC}) \bmod 86400$$

where

$$\Delta t_{UTC} = \Delta t_{LS} + A0 + A1(t_{GPS} - t_{ot} + (WN - WN_t).604800)$$

2. The present time is in the interval [DN+3/4,DN+5/4]
   in is case UTC computes to:

$$t_{UTC} = W \bmod (86400 + \Delta t_{LSF} - \Delta t_{LS})$$

where

$$W = (t_{GPS} - \Delta t_{UTC} - 43200) \bmod 86400 + 43200$$

Resolving the Time Reference Difference

In this method, different receiver clocks offsets are introduced with respect to GPS and GLONASS system time. These two clock offsets are instantaneously determined at each observation epoch together with the three unknowns of the receiver's position.

Starting with a simplified non-linear observation equation for a pseudorange observation to a satellite S of an arbitrary system (GPS or GLONASS) at an observer R,

$$PR^S_R = \rho^S_R - c.\ \delta t_R + c.\ \delta t^s$$

By taking the Taylor series expansion around P_0 ,we obtain the linearized eqn:

$$PR^S_R = \rho^S_R + (x_0 - x^S)/\ \rho^S_0 .\ (x_R - x_0)\ + (y_0 - y^S)/\ \rho^S_0 .\ (y_R - y_0) + (z_0 - z^S)/\ \rho^S_0\ .\ (z_R - z_0) + c.\ \delta t_R - c.\ \delta t^S \quad ..(1)$$

where
x_0 , y_0, z_0 being the coordinates of the approximate position and ρ^S_0

where
$\delta t_R = t_R - t_{sys}$;
t_{sys} being the GPS or GLONASS

For GPS and GLONASS observations eqn (1) becomes

$$PR^{GPSi}_R = \rho^{GPSi}_R + (x_0 - x^{GPSi})/\ \rho^{GPSi}_0 .\ (x_R - x_0)\ + (y_0 - y^{GPSi})/\ \rho^{GPSi}_0 .\ (y_R - y_0) + (z_0 - z^{GPSi})/\ \rho^{GPSi}_0 .\ z_R - z_0) + c.\ \delta t_{R,GPS} - c.\ \delta t^{GPSi} \quad ....(2)$$

$$PR^{GLOi}_R = \rho^{GLOi}_R + (x_0 - x^{GLOi})/\ \rho^{GLOi}_0 .\ (x_R - x_0)\ + (y_0 - y^{GLOi})/\ \rho^{GLOi}_0 .\ (y_R - y_0) + (z_0 - z^{GLOi})/\rho^{GLOi}_0 .\ z_R - z_0) + c.\ \delta t_{R,GLO} - c.\ \delta t^{GLOi} \quad ....(3)$$

Observations for GPS and GLONASS satellites, in matrix notation can be written as

$$\hat{u} = A.\ \hat{y}$$

$$\hat{y} = \begin{pmatrix} PR^i_R - \rho^i_0 + c.\ \delta t^i \\ PR^j_R - \rho^j_0 + c.\ \delta t^j \\ PR^k_R - \rho^k_0 + c.\ \delta t^k \\ \cdot \\ \cdot \end{pmatrix}$$

the vector of known values

$$A = \begin{pmatrix} (x_0 - x^i)/\rho^i_0 & (y_0 - y^i)/\rho^i_0 & (z_0 - z^i)/\rho^i_0 & 1 & 0 \\ (x_0 - x^j)/\rho^j_0 & (y_0 - y^j)/\rho^j_0 & (z_0 - z^j)/\rho^j_0 & 0 & 1 \\ (x_0 - x^k)/\rho^k_0 & (y_0 - y^k)/\rho^k_0 & (z_0 - z^k)/\rho^k_0 & 1 & 0 \\ & & \cdot & & \\ & & \cdot & & \end{pmatrix}$$

the design matrix (K being a GPS satellite), and

$$\hat{y} = \begin{pmatrix} (x_R - x_0) \\ (y_R - y_0) \\ (z_R - z_0) \\ c.\ \delta t_{R,GPS} \\ c.\ \delta t_{R,GLO} \end{pmatrix} \begin{matrix} = \\ = \\ = \\ = \\ = \end{matrix} \begin{pmatrix} (x_R - x_0) \\ (y_R - y_0) \\ (z_R - z_0) \\ c.\ (t_{R,} - t_{GPS}) \\ c.(\ t_R - t_{GLO}) \end{pmatrix}$$

the vector of the unknowns.

This system equation can then be solved using Least squares adjustment or Kalman filtering. The solution for these equations is only possible, if indeed there are observations to satellite of both GPS and GLONASS. Furthermore , if all but one observed satellites are from one system, with only one satellite from the second system, this additional observation contributes only to the second receiver clock offset, but does not influence the computed position.

Combining Coordinate Frames

In GLONASS only solution, satellite position in PZ-90 are obtained from the ephemeris data, thus the user position is in PZ-90. In GPS only positioning solution, the positions are given in WGS-84 and thus the user position is in WGS-84. But due to the differences in reference frame realizations, in a combined position solution, with some of the satellite coordinates in WGS-84 and some of them in PZ-90, the coordinate frame of the calculated user position in undefined. In order to get meaningful results when combining GPS and GLONASS measurements, those coordinate differences must be accounted for. A straight-forward way to accomplish this is to transform the obtained satellite coordinates at the time of signal transmission from one coordinate frame to another, before forming the design matrix and calculating the user position. Since GPS navigation has become the standard in Western countries and WGS-84 therefore is more widely spread and better-known than PZ-90, it is considered best to transform GLONASS satellite position from PZ-90 to WGS-84, thus user position also in WGS-84.

7-Parameter Coordination Transformation

Given the three- dimensional coordinates of a point P in a Cartesian coordinate frame (u,v,w), the coordinate of this point in a different, but nearly parallel coordinate frame (x,y,z) can be computed using the relation:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \quad (1+\delta s) \begin{pmatrix} 1 & \delta\omega & -\delta\Psi \\ \delta\omega & 1 & \delta\varepsilon \\ \delta\Psi & -\delta\varepsilon & 1 \end{pmatrix}$$

where

Δx , Δy, Δz coordinates of the origin of frame (u,v,w) in frame (x,y,z)

$\delta\omega$,$\delta\Psi$,$\delta\varepsilon$ differential rotations around the axes (u,v,w), respectively, to establish parallelism with frame (x,y,z).

δs differential scale change

Estimation of Transformation Parameters

The different methods of determination of transformation parameters are available in literature. The transformation parameters then were derived from comparing the coordinates in PZ-90 and WGS84. The points used for comparison could be located on the surface of the Earth or in space (satellites).

However, for observation sites given in WGS84 tracking GLONASS satellites, transformation parameters can also be determined directly from the range measurements frame. The principle of direct determination of the transformation parameters is shown for station coordinates in WGS84. It can, however be applied to any other ECEF coordinate frame as well.

The (simplified) pseudorange observation eqn from receiver R to satellite S is given by

$$PR^S_R = \rho^S_R + c.\,\delta t_R - c.\delta t^s + c.\delta^t_s{}^{Trop}{}_R + c.\delta^t_s{}^{Iono}{}_R \qquad \text{........ (a)}$$

$$\rho^S_R = \sqrt{(x_R - x^S)^2 + (y_R - y^S)^2 + (z_R - z^S)^2} \qquad \text{....... (b)}$$

Correction of Earth rotation:

$\tilde{a}^S{}_{WGS(tTX)}$ **to** $\tilde{a}^S{}_{WGS(tRX)}$

Transformation from PZ-90 to WGS84:

$\tilde{a}^S{}_{PZ(tTX)}$ **to** $\tilde{a}^S{}_{WGS(tTX)}$

Correction of Earth rotation

While the satellite signal is traveling towards the observer, the Earth- and along with it Earth fixed coordinate frame- keep rotating. During this signal travel time, it rotates by an angle of $\alpha = \rho^S_R/c.\omega_E$. This is a positive rotation around the z-axis. Thus, the satellite coordinates transform by

$$\tilde{a}^S_{PZ(tTX)} = \begin{pmatrix} 1 & \alpha & 0 \\ -\alpha & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tilde{a}^S_{PZ(tRX)}$$

.... **(c)**

for small angles α.

The geometrical distance therefore becomes

$$\rho^S_R = \sqrt{(x_{R,PZ(tTX)} - x^S_{PZ(tTX)})^2 + (y_{R,PZ(tTX)} - y^S_{PZ(tTX)})^2 + (z_{R\,PZ(tTX)} - z^S_{PZ(tTX)})^2}$$

...... **(d)**

On solving eqn. (c) and (d) the final eqn. which gives the distance between satellite and observer, corrected for Earth rotation during the signal traval time, can be written as:

$$\rho^S_R = -\omega_E/c\,.(x_{R,PZ(tRX)}\, y^S_{PZ(tTX)} - y_{R,PZ(tTX)}\, x^S_{PZ(tTX)})^2 + \Big[(\omega_E/c)^2 \big(x_{R,PZ(tRX)}\, y^S_{PZ(tTX)} - y_{R,PZ(tTX)}\, x^S_{PZ(tTX)}\big)^2 + \big(x_{R,PZ(tRX)} - x^S_{PZ(tTX)} \big)^2 + \big(y_{R,PZ(tTX)} - y^S_{PZ(tTX)}\big) + \big(z_{R,PZ(tTX)} - z^S_{PZ(tTX)} \big)$$

...... **(d)**

Of course eqn (d) is also valid in WGS84 coordinate frame:

$$\rho^S_R = -\omega_E/c\,.(x_{R,WGS(tRX)}\, y^S_{WGS(tTX)} - y_{R,WGS(tTX)}\, x^S_{WGS(tTX)})^2 + \Big[(\omega_E/c)^2 \big(x_{R,WGS(tRX)}\, y^S_{WGS(tTX)} - y_{R,WGS(tTX)}\, x^S_{WGS(tTX)}\big)^2 + \big(x_{R,WGS(tRX)} - x^S_{WGS(tTX)} \big)^2 + \big(y_{R,WGS(tTX)} - y^S_{WGS(tTX)}\big) + \big(z_{R,WGS(tTX)} - z^S_{WGS(tTX)} \big)$$

...... **(e)**

Transformation of a position vector from PZ-90 to WGS84:

$$x^S_{WGS(tTX)} = \Delta x + (1+\delta s)\,.\,(x^S_{PZ(tTX)} + \omega\delta\,.\,y^S_{PZ(tTX)} - \Psi\delta.\,z^S_{PZ(tTX)})$$
$$y^S_{WGS(tTX)} = \Delta y + (1+\delta s)\,.\,(-\omega\delta.\,x^S_{PZ(tTX)} + y^S_{PZ(tTX)} + \varepsilon\delta.\,z^S_{PZ(tTX)})$$
$$z^S_{WGS(tTX)} = \Delta z + (1+\delta s)\,.\,(\Psi\delta.x^S_{PZ(tTX)} - \varepsilon\delta\,.\,y^S_{PZ(tTX)} + z^S_{PZ(tTX)})$$

.........**(f)**

Inserting eqn (f) in eqn (d) gives the geometrical range with it eqn (a) is non-linear in the unknown transformation parameters. Before trying to solve a system of observation eqns, the eqn (d) has to be linearized using Taylor series expansion around the approximate values of Δx_0, Δy_0, Δz_0, δs_0, $\delta\varepsilon_0$, $\delta\Psi_0$, $\delta\omega_0$.

The above mentioned solution for transformation parameters were estimated for number of days using Kalman filter. These daily solutions were averaged to obtain a set of transformation parameters:

Parameter	Δx[m]	Δy[m]	Δz[m]	$\delta s[10^{-9}]$	$\delta\varepsilon\ [10^{-6}]$	$\delta\Psi\ [10^{-6}]$	$\delta\omega\ [10^{-6}]$
Value	**0.404**	**0.357**	**-0.476**	**-2.614**	**0.118**	**-0.058**	**-1.664**
Std. Dev.	**1.039**	**1.147**	**0.456**	**63.860**	**0.090**	**0.112**	**0.170**

Daily solution using the transformation introduced above were close to the solutions using the transformation given in (*Roßbach et al.,1996*).

Pseudorange Measurements computation for GPS/GLONASS system

Once the signal travel time and the satellite position at the time of signal transmission are known, the receiver position can be computed just as with GPS by linearization equations and solving for the unknowns. There are four unknowns, namely the x-,y-,z-coordinates of the users position and the receiver clock offset w.r.t GLONASS system time. Thus, to solve for these four unknowns, measurements to at least four satellites are necessary.

Analogously to GPS, the pseudorange observation equation from observer R to satellite S can be written as:

$$PR^{S}_{R} = \rho^{S}_{R} + c.\ \delta t_R - c.\delta t^{s} + c.\delta t^{s}_{R}{}^{Trop}{}_{R} + c.\delta t^{s}_{R}{}^{Iono}{}_{R} + c\ L^{S}_{R} + \varepsilon^{S}_{R} \ ......(g)$$

Due to the different frequencies involved, the signals of different GLONASS satellites will take different path in the receiver. These different paths may leads to different hardware delays for signal from satellites. These different delays are modeled by the L^{S}_{R} term in eqn.(g), these delays are treated as dependent only on the satellite in eq (g).
Splitting this hardware dalay into a common term and a satellite (channel) dependent bias:

$$L^{S}_{R} = L_{R,GLO} + \delta t^{s}_{R,ICB}$$

$$PR^{S}_{R} = \rho^{S}_{R} + c.\ \delta t_R - c.\delta t^{s} + c.\delta t^{s}_{R}{}^{Trop} + c.\delta t^{s}_{R}{}^{Iono} + c\ (L_{R,GLO} + \delta t^{s}_{R,ICB}) + \varepsilon^{S}_{R} \ ...(h)$$

this common delay $L_{R,GLO}$ can no longer be seprated from the clock term δt^{s}_{R} and the satellite dependent bias $\delta t^{s}_{R,ICB}$ is called inter-channel bias.

on linearization of eqn (h) the observation eqn transform to

$$PR^{S}_{R} - \rho^{S}_{R} - c.(\ \delta t_{R,0} + L_{R,GLO,0}) + c.\delta t^{s} - c.\delta t^{s}_{R}{}^{Trop} - c.\delta t^{s}_{R}{}^{Iono} = (x_0 - x^{S})/\ \rho^{S}_{0}.\ (x_R - x_0) + (y_0 - y^{S})/\ \rho^{S}_{0}.\ (y_R - y_0) + (z_0 - z^{S})/\ \rho^{S}_{0}.\ z_R - z_0) + c.[(\delta t_R + L_{R,GLO,0})]\ ) + \varepsilon^{S}_{R}$$

where known and modeled values have been shifted to the left-hand side of the equation. Having measurement for number of satellites 1,2,3……,n, one can summarize the result set of observation equation in matrix notation:

$$\hat{u} = B.\,\hat{y} + \varepsilon$$

where

$$\hat{u} = \begin{pmatrix} PR^1_R - \rho^1_0 - c.(\delta t_{R,0} + L_{R,GLO,0}) + c.\,\delta t^1 - c.\delta t^{1\ \ Trop}_R - c.\delta t^{1\ \ Iono}_R \\ PR^1_R - \rho^1_0 - c.(\delta t_{R,0} + L_{R,GLO,0}) + c.\,\delta t^1 - c.\delta t^{1\ \ Trop}_R - c.\delta t^{1\ \ Iono}_R \\ \vdots \\ PR^n_R - \rho^n_0 - c.(\delta t_{R,0} + L_{R,GLO,0}) + c.\,\delta t^n - c.\delta t^{n\ \ Trop}_R - c.\delta t^{n\ \ Iono}_R \end{pmatrix}$$

the vector of the known values,

$$B = \begin{pmatrix} (x_0 - x^1)/\rho^1_0 & (y_0 - y^1)/\rho^1_0 & (z_0 - z^1)/\rho^1_0 & 1 \\ (x_0 - x^2)/\rho^2_0 & (y_0 - y^2)/\rho^2_0 & (z_0 - z^2)/\rho^2_0 & 1 \\ & \vdots & & \\ (x_0 - x^n)/\rho^n_0 & (y_0 - y^n)/\rho^n_0 & (z_0 - z^n)/\rho^n_0 & 1 \end{pmatrix}$$

the design matrix,

$$\hat{y} = \begin{pmatrix} (x_R - x_0) \\ (y_R - y_0) \\ (z_R - z_0) \\ c.[(\delta t_R + L_{R,GLO}) - (.[(\delta t_{R,0} + L_{R,GLO,0}) \end{pmatrix}$$

the vector of the unknowns, and

$$\varepsilon = \begin{pmatrix} \varepsilon^1_R \\ \varepsilon^2_R \\ \cdot \\ \cdot \\ \varepsilon^n_R \end{pmatrix}$$

the noise vector.

This system of eqns can then be solved using the conventional methods e.g least square adjustment or Kalman filtering.

Since the satellite positions as computed from GLONASS ephemeris (or almanac) data are expressed in the PZ-90 frame, the resulting receiver position is also given in this coordinate frame. To get the position in a different coordinate frame, the resulting coordinates must be transformed as desired.

Summary

Both GPS and GLONASS are very similar systems. However, in all similarity, there are also differences between these teo systems. These differences, and how they effect the combined evaluation of GPS and GLONASS satellite observation, have been worked out. The first of these differences is the different frame for time used by GPS and GLONASS. The difference between these two time frames ins not known in real-time. However, this problem can be easily overcome by introducing the offset between the system times as an additional unknown in the observation equation. This means scarifying one observation to solve for that additional parameter. But this is not a problem as long as the number of additional satellites are greater than one.

The next difference is the different coordinate reference frame used by GPS(WGS84) and GLONASS(PZ-90).this difference can be overcome by converting GLONASS satelliete position from the PZ-90 frame to WGS84 frame before using them in a combined positioning solutin. This conversion is done by means of seven parameter Helmert transformation.

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