Analytical Gyrocompass Alignment of Strapdown INS on a Stationary Base

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ABSTRACT

The paper describes the technique of automatic alignment of Strap Down (SD) Inertial Navigation Systems (INS) on an oscillatory base. The technique known as the Gyrocompass Alignment is entirely self-contained. Initial alignment is a necessary and essential step in Strap Down Inertial Navigation. The paper introduces the problem of alignment as defined by Wahba and develops the entire theory relevant to Gyrocompass Alignment from first principles. The derived algorithms are put to use in aligning indigeneous Dynamically Tuned Gyro (DTG) & Ring Laser Gyro (RLG) based Strap Down Inertial Navigation systems. A two stage self-alignment scheme generally called Coarse & Fine alignment are explored. Coarse alignment computes initial transformation matrix relating the body and reference frame. Fine alignment stage corrects the initial transformation matrix by correction signal, which is computed by using estimates of the error angle between reference frame and the corresponding computed frame. A linear kalman filter is used to implement and an error analysis is presented.

Keywords: Strapdown Navigation, Alignment, Gyrocompass, Kalman filter.

1. INTRODUCTION

Initial misalignment is one of the major error sources of Strap Down Inertial Navigation Systems(SDINS). Initial misalignment errors translate into incorrect attitude, position and velocity information for all future time. The aim of this paper is to estimate the elements of C_b^{ned} with measurements only from the SDINS. This paper employs an in-vogue technique known as analytical gyrocompass alignment for the purpose. Analytical gyrocompass alignment is the only such technique that requires no external aiding. The SDINS being popular for its self-contained anti-jamming, environment independent functioning, it becomes necessary and obvious that its alignment also be self contained, and hence analytical gyrocompass alignment. This paper describes in detail with complete mathematical treatment, the art of *Analytical Gyrocompass Alignment*.

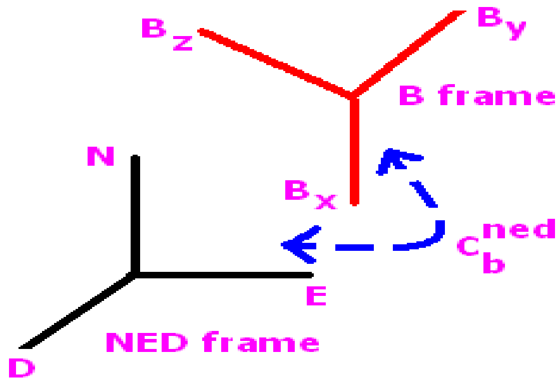

Figure 1: NED to Body frame conversion and vice versa

1.1 INS Error Sources

INS errors are due to

- initial misalignment
- inertial sensor errors

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- computational inaccuracies

The numerical accuracy of computations is sufficiently enhanced due to the advances in modern day computers. The computational errors, hence, do not sufficiently affect the INS to warrant any corrective action. Inertial sensor errors are inherent imperfections that creep into the INS either at the time of manufacture of the INS or arise due to the random influences that affect the INS during operation. While the manufacturing defects could be rectified in certain cases(eg.skew misalignment and calibration), other manufacturing errors and random errors are beyond any closed form mathematical description. They have to be handled based on the statistical nature of their error build up.

For dead reckoning systems like the INS, any lack of precision in the initialization, will have catastrophic effect on the navigation performance and accuracy. Initial misalignment error will result in error in velocity and position with linear and quadratic growth in time respectively.[5] Hence the acute need for precise initial alignment.

2. GYROCOMPASS ALIGNMENT

2.1 Strapdown Gyrocompass Alignment

Gyrocompass alignment for strapdown systems is a process of virtual alignment since no actual rotation takes place to align the body frame with the navigation frame. The DCM gets estimated while the SDINS is essentially stationary. Gyrocompass alignment is comprised of a initial coarse estimation and later finer alignment with a Kalman filter. Coarse alignment estimates the DCM within 1^0 of error of the true DCM. The error seperating the coarse DCM with the true DCM is known as the misalignment angle (ψ angle). The fine alignment employs a Kalman filter to estimate the misalignment angles and corrects them to give the true DCM. Gyrocompass alignment does not work near the poles as the North and Down axes coincide each other.

2.1.1 Analytical coarse alignment

The DCM is a 3×3 matrix with 9 unknowns. DCM could be constructed from 3 euler angles. Though there are lesser number of unknowns in the euler angle representation, they could not be used for Gyrocompass alignment due to the gimbal lock problem. When the pitch euler angle is 90^0, the roll and yaw euler angles are indeterminate.

To solve for the DCM, from the linear algebraic point of view, it is required that 9 equations be available to solve for the 9 unknowns. The measurement of a vector both in body and *ned* frames yields 3 simultaneous equations. Let σ be any vector in three dimensional space, then :

$$\sigma^{ned} \; = \; C_b^{ned} \, \sigma^b$$

forms 3 simultaneous equations. Hence, to get a closed form expression for C_b^{ned}, three vectors should be simultaneously determined in the body as well as in the *ned* frame, giving a complete system of 9 equations with 9 unknowns. Three methods are described hereunder to obtain an estimate of C_b^{ned}. The coarse estimate $\hat{C}_b^{ned}$ is away from true C_b^{ned} due to accelerometer and gyro errors.

Method 1 The actual apparent gravity vector(including centripetal acceleration), Earth rate vector and the cross product of the two vectors in the body coordinates and in the *ned* frame form are related as follows -

$$\begin{aligned} g^b &= \hat{C}_{ned}^b \, g^{ned} \\ \omega_{ie}^b &= \hat{C}_{ned}^b \, \omega_{ie}^{ned} \\ g^b \times \omega_{ie}^b &= \hat{C}_{ned}^b \, \left(g^{ned} \times \omega_{ie}^{ned}\right) \end{aligned} \tag{1}$$

Taking the transpose of both sides and combining the three equations into one matrix equation yields

$$\begin{pmatrix} {g^b}^T \\ {\omega_{ie}^b}^T \\ \left(g^b \times \omega_{ie}^b\right)^T \end{pmatrix} = \begin{pmatrix} {g^{ned}}^T \; \hat{C}_b^{ned} \\ {\omega_{ie}^{ned}}^T \; \hat{C}_b^{ned} \\ \left(g^{ned} \times \omega_{ie}^{ned}\right)^T \; \hat{C}_b^{ned} \end{pmatrix} \tag{2}$$

Taking the DCM $\hat{C}_b^{ned}$ on the LHS, eqn.2 becomes -

$$\hat{C}_b^{ned} = \begin{pmatrix} {g^{ned}}^T \\ {\omega_{ie}^{ned}}^T \\ \left(g^{ned} \times \omega_{ie}^{ned}\right)^T \end{pmatrix}^{-1} \begin{pmatrix} {g^b}^T \\ {\omega_{ie}^b}^T \\ \left(g^b \times \omega_{ie}^b\right)^T \end{pmatrix} \tag{3}$$

Method 2 An other orthogonal frame of reference could be constructed using the three vectors g^{ned}, $g^{ned} \times \omega_{ie}^{ned}$ and $\left(g^{ned} \times \omega_{ie}^{ned}\right)$.

$$\hat{C}_b^{ned} = \begin{pmatrix} {g^{ned}}^T \\ \left(g^{ned} \times \omega_{ie}^{ned}\right)^T \\ \left(\left(g^{ned} \times \omega_{ie}^{ned}\right) \times g^{ned}\right)^T \end{pmatrix}^{-1} \begin{pmatrix} {g^b}^T \\ \left(g^b \times \omega_{ie}^b\right)^T \\ \left(\left(g^b \times \omega_{ie}^b\right) \times g^b\right)^T \end{pmatrix} \tag{4}$$

Method 3 The gravity vector points vertically downwards. Due to Earth's rotation, ω_{ie}, the gravity vector is deflected to the left in the inertial space. The vector of the gravity gradient thus formed is perpendicular to the gravity vector, and together with their cross product form an orthogonal frame of reference in the NED coordinate reference frame. This ingenious method due to Bellantoni,[4] is considered to be the most accurate of the analytical coarse alignment methods in vogue, since it considers the relative gradient of the gravity vector as compared to absolute Earth's angular velocity. The gravity gradient method minimizes the effects of gyro drift.

$$\hat{C}_b^{ned} = \begin{pmatrix} {g^{ned}}^T \\ \left(\dot{g}^{ned}\right)^T \\ \left(g^{ned} \times \dot{g}^{ned}\right)^T \end{pmatrix}^{-1} \begin{pmatrix} {g^b}^T \\ \left(\dot{g}^b\right)^T \\ \left(g^b \times \dot{g}^b\right)^T \end{pmatrix} \tag{5}$$

The gradient of the gravity vector in the inertial space is easily obtained as follows -

$$\begin{aligned} \dot{g}^b &\cong C_I^b \frac{\Delta g^I}{\Delta t} \\ \Delta g^b(t, t_0) &\cong C_I^b \left[g^I(t) - g^I(t_0)\right] \end{aligned}$$

Expressing $g^I(t)$ and $g^I(t_0)$ in the body frame measured accelerations-

$$\begin{aligned} \Delta g^b(t, t_0) &\cong C_I^b(t) \left[C_b^I(t) g^b(t) - C_b^I(t_0) g^b(t_0)\right] \\ &\cong g^b(t) - C_b^b(t, t_0) g^b(t_0) \end{aligned}$$

$C_b^b(t, t_0)$ is the transformation matrix relating body frame at instant t_0 to instant t. Gravity gradient $\dot{g}^{ned}$ is obtained similarly.

Measurables in Body frame Sitting on the earth the strapdown system measures the specific force of the Earth pushing upward, while the apparent acceleration due to gravity is downward, hence

$$g^b = \begin{pmatrix} -f^{xb} \\ -f^{yb} \\ -f^{zb} \end{pmatrix} \omega_{ie}^b = \begin{pmatrix} \omega^{xb} \\ \omega^{yb} \\ \omega^{zb} \end{pmatrix}$$

where $\omega^{xb}, \omega^{yb}, \omega^{zb}$ and f^{xb}, f^{yb}, f^{zb} are the angular velocities and accelerations sensed by the gyroscopes and accelerometers respectively expressed in body coordinates.

Measurables in local frame The apparent acceleration due to gravity and the rate of Earth's rotation in the NED frame are given by -

$$g^{ned} = \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}$$

$$\omega_{ie}^{ned} = \begin{pmatrix} \omega^N \\ \omega^E \\ \omega^D \end{pmatrix} = \begin{pmatrix} \omega \cos L \\ 0 \\ -\omega \sin L \end{pmatrix}$$

where

- g - Modeled apparent gravity value at the strapdown location (after subtracting the centripetal acceleration due to earths rotation). At INS location, g is found to be 9.783117548 m/sec^2.
- ω - Angular velocity of Earth: $7292115167 \times 10^{-14}$ rad/sec or $15.041067^0/h$ as per WGS 84 standards.
- L - local latitude at the INS location 17.2539^0.

2.1.2 Fine alignment

The procedure of coarse alignment gives a fair estimate of the direction cosine matrix relating the body to NED frame(within a degree of accuracy). Coarse alignment actually measures roll and pitch angles with very great degree of accuracy of the order of 0.01^0 or better. The yaw angle however is accurate upto 1^0. High accuracy in roll and pitch angles estimation is due to the greater observability of north and east accelerometers, while the yaw accuracy is limited due to the poor observability of the east gyro.

This section presents the dynamics of static SDINS convenient for processing with a Kalman filter. Kalman filter is used to predict the misalignment angles from noisy North and East accelerometer(s) and East gyro measurements. The local level accelerometers and the east gyro measure null magnitude irrespective of the ambiguity in location. Hence they are chosen as observables. The Kalman filter is said to be *converged* when the misalignment angles are converged.

This section describes suitable error models in terms of initial misalignment angles.

2.2 ψ angle error model

2.2.1 ψ angle error dynamics

The derivation of the non-linear large ψ angle error dynamics is given in the works of Wienred and Durrant Whyte. They are rederived here for comprehensiveness. The derivative of the DCM that transforms b-frame to c-frame is given by

$$\dot{C}_b^c \;=\; C_b^c\, \Omega_{ib}^b \;-\; \Omega_{ic}^c\, C_b^c \tag{6}$$

The derivative of the DCM formed using measured gyro rates $\hat{\Omega}_{ib}^b$ is

$$\dot{C}_b^p \;=\; C_b^p\, \hat{\Omega}_{ib}^b \;-\; \Omega_{ic}^c\, C_b^p \tag{7}$$

where $\hat{\Omega}_{ib}^b$ is measured body frame angular velocities, with

$$\hat{\Omega}_{ib}^b \;=\; \Omega_{ib}^b \;+\; \in^b \tag{8}$$

$$\begin{aligned}
\text{Let}\quad \Delta C &= C_b^p - C_b^c,\ \text{then}\\
&\Rightarrow C_b^p \;-\; C_p^c\, C_b^p\\
&\Rightarrow \left(I \;-\; C_p^c\right)\, C_b^p\\
\text{and}\quad \dot{\Delta C} &= \left(I \;-\; C_p^c\right)\, \dot{C}_b^p \;-\; \dot{C}_p^c\, C_b^p\\
&\Rightarrow C_b^p\, \hat{\Omega}_{ib}^b \;-\; \Omega_{ic}^c\, C_b^p \;-\; C_p^c\, C_b^p\, \hat{\Omega}_{ib}^b \;+\; C_p^c\, \Omega_{ic}^c\, C_b^p \;-\; \dot{C}_p^c\, C_b^p
\end{aligned} \tag{9}$$

$\dot{\Delta C}$ can also be obtained from $\Delta C \;=\; C_b^p \;-\; C_b^c$:

$$\begin{aligned}
\dot{\Delta C} &= \dot{C}_b^p \;-\; \dot{C}_b^c\\
&\Rightarrow C_b^p\, \Omega_i^b b \;-\; \Omega_{ic}^c\, C_b^p \;-\; C_b^c\, \Omega_{ib}^b \;+\; \Omega_{ic}^c\, C_b^c\\
&\Rightarrow C_b^p\, \Omega_i^b b \;-\; \Omega_{ic}^c\, C_b^p \;-\; C_p^c\, C_b^p\, \Omega_{ib}^b \;+\; \Omega_{ic}^c\, C_p^c\, C_b^p
\end{aligned} \tag{10}$$

Equating eqn.9 and eqn.10 we get :

$$\dot{C}_p^c\, C_b^p \;+\; C_p^c\, C_b^p\, \hat{\Omega}_{ib}^b \;-\; C_p^c\, \Omega_{ic}^c\, C_b^p - C_p^c\, C_b^p\, \Omega_{ib}^b \;+\; \Omega_{ic}^c\, C_p^c\, C_b^p \;=\; 0 \tag{11}$$

Noting $\hat{\Omega}_{ib}^b \;=\; \Omega_{ib}^b \;+\; \in^b$ and right multiplying eqn.11 with C_p^b :

$$\dot{C}_p^c \;+\; C_p^c\, C_b^p\, \in^b\, C_p^b \;-\; C_p^c\, \Omega_{ic}^c \;+\; \Omega_{ic}^c\, C_p^c \;=\; 0 \tag{12}$$

Since $\in^p \;=\; C_b^p\, \in^b\, C_p^b \;=\; C_c^p\, \in^c\, C_p^c$, and $C_p^c \in^p \;=\; \in^c\, C_p^c$, eqn.12 is simplified as :

$$\dot{C}_p^c \;+\; \in^c\, C_p^c - C_p^c\, \Omega_{ic}^c \;+\; \Omega_{ic}^c\, C_p^c \;=\; 0 \tag{13}$$

Writing $\dot{C}_p^c \;=\; C_p^c\, \Omega_{cp}^p$ and left multiplying with C_c^p, eqn.13 is written as :

$$\begin{aligned}
& C_p^c\, \Omega_{cp}^p \;+\; \in^c\, C_p^c \;-\; C_p^c\, \Omega_{ic}^c \;+\; \Omega_{ic}^c\, C_p^c &= 0\\
\Rightarrow\;& \Omega_{cp}^p \;+\; C_c^p\, \in^c\, C_p^c \;-\; \Omega_{ic}^c \;+\; C_c^p\, \Omega_{ic}^c\, C_p^c &= 0
\end{aligned} \tag{14}$$

$C_c^p \in^c C_p^c = \in^p$ and $C_c^p \, \Omega_{ic}^c \, C_p^c = \Omega_{ic}^p$, eqn.14 reduces to

$$\Omega_{cp}^p + \in^p - \Omega_{ic}^c + \Omega_{ic}^p = 0 \quad (15)$$

The vectorized form of eqn.15 is represented as :

$$\omega_{cp}^p + \epsilon^p - \omega_{ic}^c + \omega_{ic}^p = 0 \quad (16)$$

Since $\omega_{ic}^p = C_c^p \, \omega_{ic}^c$, eqn.16 can be written as

$$\omega_{cp}^p = (I - C_c^p) \, \omega_{ic}^c - \epsilon^p \quad (17)$$

The angular velocity of the c frame with respect to p frame is the rate of change of the ψ angle separating the two frames. Thus $\omega_{cp}^p = \dot{\psi}$.

$$\dot{\psi} = (I - C_c^p) \, \omega_{ic}^c - \epsilon^p \quad (18)$$

For small misalignment between the c and p frames,

$$C_c^p = \begin{bmatrix} 1 & \psi^d & -\psi^e \\ -\psi^d & 1 & \psi^n \\ \psi^e & -\psi^n & 1 \end{bmatrix}$$

For small psi-angle error model, eqn.18 can be written as

$$\begin{aligned} \dot{\psi} &= \psi \times \omega_{ic}^c - \epsilon^p \quad \text{(or)} \\ \dot{\psi} + \omega_{ic}^c \times \psi &= -\epsilon^p \\ \dot{\psi} + \omega_{in}^n \times \psi &= -\epsilon^p \end{aligned} \quad (19)$$

Measurement Model

Fine alignment phase takes the established $\hat{C}_b^{ned}$ from coarse alignment and refines it with a Kalman filter. The coarse DCM estimate, $\hat{C}_b^{ned}$ is used to transform the sensor measurements in the body frame to the *ned* frame. The North and East velocity measurements are used for leveling and the angular velocity in the East is used for heading refinement. Using the coarse estimate of the DCM $\hat{C}_b^{ned}$, the body frame accelerations are converted to the *ned* frame as-

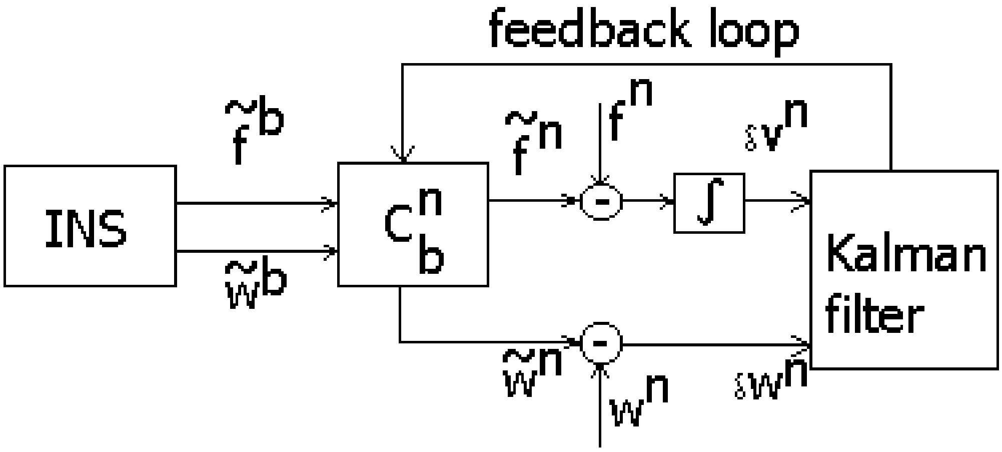

Figure 2: Fine alignment with δv^n & $\delta \omega^n$ measurements

$$f^{ned} = \hat{C}_b^{ned} \, f^b \quad (20)$$

The *ned* frame specific force error in measurement is given by

$$\begin{aligned} \delta f^n &= \hat{C}_b^{ned} \widetilde{f}^b - f^n \\ &= C_b^{ned} (I + \Psi) (f_b + \delta f^b) - f^n \\ &\approx C_b^{ned} f^b + \Psi C_b^{ned} f^b + C_b^{ned} \delta f^b - f^n \\ &= \Psi C_b^{ned} f^b + C_b^{ned} \delta f^b \\ &= (f^n \times)(\psi^N \ \psi^E \ \psi^D)^T + C_b^{ned} \delta f^b \end{aligned} \quad (21)$$

or error in velocity is obtained by integrating the specific force error as shown in fig.2. The error in velocity is given by

$$\delta v^n = \int \left(\hat{C}_b^{ned} \tilde{f}^b - f^n \right) dt \tag{22}$$

and similarly the angular velocity measurement error is given by

$$\begin{aligned} \delta\omega^n &= \hat{C}_b^{ned}\tilde{\omega}^b - \omega^n \\ &= C_b^{ned}(I + \Psi)(\omega^b + \delta\omega^b) - \omega^n \\ &\approx C_b^{ned}\omega^b + \Psi C_b^{ned}\omega^b + C_b^{ned}\delta\omega^b - \omega^n \\ &= \Psi C_b^{ned}\omega^b + C_b^{ned}\delta\omega^b \\ &= (\omega^n \times)(\psi^N \ \psi^E \ \psi^D)^T + C_b^{ned}\delta\omega^b \end{aligned} \tag{23}$$

The measured observables from eqn.22 are directly related with the state variables δv^N and δv^E. The error in angular velocity in the East direction is related with the ψ misalignment angles. The measurement matrix is given as follows :

$$\begin{aligned} \mathbf{z} &= \mathbf{H}\,\mathbf{x} + \Gamma(t), \quad \Gamma(t) \sim N(0, R) \\ \mathbf{H} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\omega sinL & 0 & -\omega cosL & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{24}$$

The number of observable states are given by

$$Rank \begin{bmatrix} \mathbf{H} \\ \mathbf{H}\,\mathbf{A_i} \\ \mathbf{H}\,\mathbf{A_i}^{\mathbf{2}} \\ \vdots \\ \mathbf{H}\,\mathbf{A_i}^{\mathbf{9}} \end{bmatrix} = 7 < 10$$

Hence the entire state vector cannot be estimated with the available observations. We follow[2] and mark δf^N, δf^E and $\delta\omega^E$ to be unobservable states.

We thus have a complete dynamical system of SDINS in state space form suitable for Kalman filter implementation.

System Dynamics Model

The present ten states could be reduced to six states by considering the north and east acceleration as the observables instead of the north and east velocities. The reduced state vector consists of the following variables

$$\left[\ \psi^N \quad \psi^E \quad \psi^D \quad \delta\omega^N \quad \delta\omega^E \quad \delta\omega^D \ \right]^T$$

and the state model is reduced as

$$\begin{bmatrix} \dot{\mathbf{x}}' \\ \dot{\mathbf{b}} \end{bmatrix} = \begin{bmatrix} \mathbf{A'}_{3\times3} & \mathbf{T_i}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \begin{bmatrix} \mathbf{x}' \\ \mathbf{b} \end{bmatrix} \tag{25}$$

where

$$A' = \begin{bmatrix} 0 & \omega^D & 0 \\ -\omega^D & 0 & \omega^N \\ 0 & -\omega^N & 0 \end{bmatrix}$$

$$T_i = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

The observability matrix is also modified to relate the horizontal acceleration errors directly to the Ψ misalignment angles

$$\mathbf{H} = \begin{bmatrix} 0 & g & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & 0 & 0 & 0 \\ -\omega sinL & 0 & -\omega cosL & 0 & 0 & 0 \end{bmatrix}$$

The reduced order model performs to the same accuracy as that of the 10-state model. Also, the feedback loop shown in fig.2 is optional. The misalignment angles need not be continously corrected during the fine alignment. They could be corrected at the end of fine alignment after Kalman filter loop is complete, with no loss of accuracy.

Kalman filter

The discrete Kalman filtering equations are given as follows:[2]

$$\left.\begin{array}{ll} \hat{\mathbf{x}}_{\mathbf{k}} & = \mathbf{A}_{\mathbf{k},\mathbf{k-1}}\ \hat{\mathbf{x}}_{\mathbf{k-1}}\ +\ \mathbf{G}_{\mathbf{k}}\left[\mathbf{z}_{\mathbf{k}}\ -\ \mathbf{H}_{\mathbf{k}}\ \mathbf{A}_{\mathbf{k},\mathbf{k-1}}\ \hat{\mathbf{x}}_{\mathbf{k-1}}\right] \\ \mathbf{G}_{\mathbf{k}} & = \mathbf{P}_{\mathbf{k}}\ \mathbf{H}_{\mathbf{k}}^{\mathbf{T}}\ \left[\mathbf{H}_{\mathbf{k}}\ \mathbf{P}_{\mathbf{k}}\ \mathbf{H}_{\mathbf{k}}^{\mathbf{T}}\ +\ \mathbf{R}_{\mathbf{k}}\right]^{-1} \\ \mathbf{K}_{\mathbf{k}} & = (\mathbf{I}\ -\ \mathbf{G}_{\mathbf{k}}\ \mathbf{H}_{\mathbf{k}})\ \mathbf{P}_{\mathbf{k}} \\ \mathbf{P}_{\mathbf{k+1}} & = \mathbf{A}_{\mathbf{k+1},\mathbf{k}}\ \mathbf{K}_{\mathbf{k}}\ \mathbf{A}_{\mathbf{k+1},\mathbf{k}}^{\mathbf{T}} + \mathbf{Q}_{\mathbf{k}} \end{array}\right\} \tag{26}$$

where $\mathbf{A}_{\mathbf{k},\mathbf{k-1}}$ is the discrete state transformation matrix.

State vector initialization

$$\left[\begin{array}{cccccc} \psi^N & \psi^E & \psi^D & \delta\omega^N & \delta\omega^E & \delta\omega^D \end{array}\right]^T = \mathbf{0}_{10x1}^T$$

Design parameters for medium accuracy SDINS are initialized as follows -

Kalman filter initialization For a medium accuracy $\approx 0.01^0/hr$ class of gyro, the initial misalignment angles ψ^N, ψ^E, ψ^D are chosen as 1^0. The constant and random drifts of each gyro are chosen as $0.02^0/h$ and $0.01^0/h$ respectively.

$$\begin{array}{rcl} \mathbf{P(0)} & = & diag\ \ \left\{(1^0)^2, (1^0)^2, (1^0)^2, \right. \\ & & \left. (0.02^0/h)^2, (0.02^0/h)^2, (0.02^0/h)^2\right\} \\ \mathbf{Q} & = & diag\ \ \left\{(0.01^0/h)^2, (0.01^0/h)^2, (0.01^0/h)^2, 0, 0, 0\right\} \\ \mathbf{R} & \approx & diag\ \ \left\{10e^{-8}; 10e^{-8}; 1e^{-16}\right\} \end{array}$$

3. RESULTS

3.1 Simulated data

The results shown in fig.3 the performance of gyrocompass alignment on data obtained from low cost Ring Laser Gyro based INS with $0.01^0/hr$ class of gyro mounted on a rate table. Test data is generated for various angles of yaw and the gyrocompass alignment procedure is tested with such data. The fig.3 shows the gyrocompass yaw values along with their true values on the rim with spoke every 45^0 apart. The accuracy in yaw is observed to be $\approx 0.2^0$.

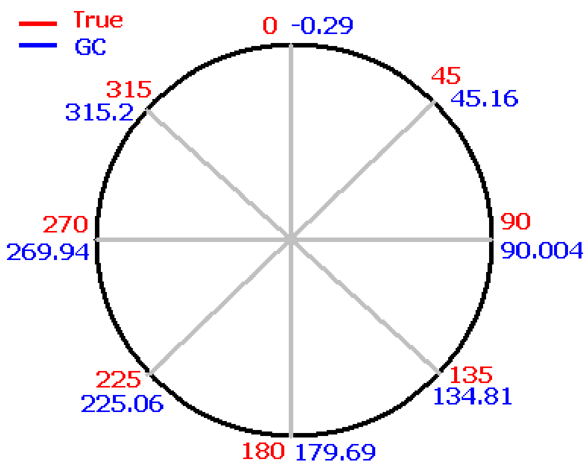

Figure 3: Yaw results on simulated data at various angles

3.2 Estimation of Misalignment

The diagrams fig.4 show the estimation of misalignment angle, with the simulation is carried out in open loop. As can be seen, the convergence in Yaw is longer and poor due to the lesser observability of east axis gyroscope.

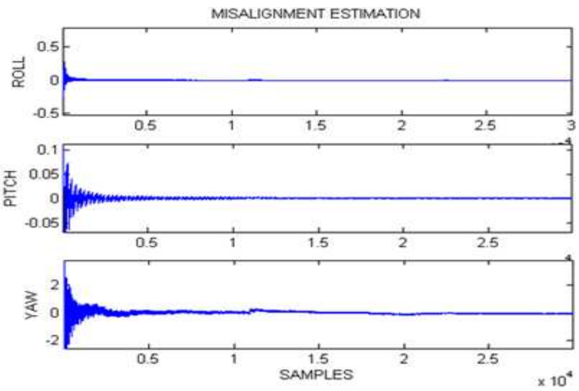

Figure 4: Misalignment Estimation Convergence Diagram

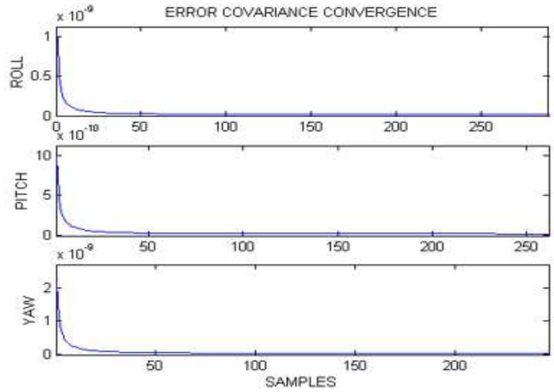

Figure 5: Error covariance convergence

3.3 Discussion

Kalman filter is said to be converged if the diagnol elements of the error covariance matrix converge in steady state. Fig. 5 shows the error convergence characteristics.

4. CONCLUSIONS

The problem of initial alignment of Strapdown Inertial Navigation Systems is solved with the Gyrocompass Alignment procedure. Three methods of computing coarse alignment matrix are elucidated. A 6 state error model is derived for small misalignment dynamics of Strapdown Inertial Navigation Systems. A Kalman filter is designed and implemented with 3 observables for the purpose of fine alignment. The error in Azimuth, Yaw, is found to be within 0.2^0 after fine alignment on medium grade INS data.

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