Sensing a differential Gouy phase in a ring laser

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ABSTRACT

A theoretical study is made to analyze the difference in Gouy phase for the clockwise and anti clockwise beams traveling in a rotating ring laser. The magnitude of this phase can be within a range of 0 to π. The treatment is a self-consistent one, in which the round trip field is equal to the original field in the cavity. The self-consistency gives conditions on the amplitude and the frequency of oscillation of the modes oppositely traveling - in terms of the parameters of the system. This model shows that, for a rotating ring laser gyro the Gouy phase adds to change in the frequency shift of the two modes. We propose a mechanism to control and measure this change.

Keywords: Gouy Phase, Ring Laser

1. INTRODUCTION

In 1890 Gouy showed that a focused electromagnetic beam acquires an additional axial 180^{o} phase shift with respect to a plane wave as it evolves through its focus [1-3]. This phase shift has important consequences. A hundred years have passed since Gouy made his discovery, and efforts are being made to provide a satisfying physical interpretation of this phase jump and to place it within the context of other phase anomalies.

Recently Gouy phase effects by placing a lens inside the Sagnac interferometer have been studied [4]. In this paper we present an unambiguous manifestation of the differential Gouy phase in a ring laser. The reason for using the ring laser for this study is that it provides increased flexibility in resonator design as compared to a standing wave cavity. In particular a ring cavity can easily employ a comparatively short beam expansion section, using readily available short focal length optical elements, and then a long collimated beam section at large beam diameter, for obtaining full power extraction from large diameter laser gain media, as explained below.

Through the use of a ring laser, phase changes in optical length may be readily observed as a difference between resonant frequencies for oppositely directed waves. This principle has potential to increase accuracy of the ring laser rotation sensor.

2. FORMALISM

For the starting, this analysis is restricted to laser source producing Gaussian beam. In general, the solution of Maxwell equations for the scalar electric field E (x, y, z) of a Gaussian beam can be written as

$$E(x,y,z)=E_0\frac{\omega_0}{\omega(z)}\exp\left\{-i\left[kz-\phi(z)\right]-r^2\left[\frac{1}{\omega^2(z)}+\frac{ik}{2R(z)}\right]\right\} \quad (1)$$

The expression describes the behavior of the laser beam amplitude as a function of the transverse coordinates x, y and the axial coordinate z. $k=2\pi/\lambda$ is the wave number, where λ is the wavelength in the material where the beam propagates.

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Here $R(z)$ is the radius of curvature of the wave front, $\omega(z)$ is the beam radius and $\phi(z)$ is the Gouy phase shift and it deserves the special attention, as it depends purely on the geometry of the cavity.

This additional phase shift can be understood mathematically as follows. The propagation for the lowest order Gaussian beam includes variation of both a spot size and a cumulative phase shift with the axial distance *z*. These are given on the optical axis (*x* = *y* = 0) by the factors

$$E(x,y,z)\ \alpha\quad \frac{\bar{q}_0 e^{-jkz}}{\bar{q}(z)} = \frac{\exp(-ikz + i\phi(z))}{\omega(z)} \qquad (2)$$

In addition to the free space or plane wave phase shift given by e^{-ikz} term, an added axially-varying phase shift $\phi(z)$ is given by

$$\phi(z) = \tan^{-1}\left(\frac{z}{z_R}\right) \qquad (3)$$

where z_R is the Rayleigh range.

The stability of the Gaussian mode system in a ring laser, the condition of the self consistency is imposed i.e., that the beam should reproduce itself in *shape*, *amplitude*, and *phase*, after each roundtrip. An arbitrary reference plane is chosen in the resonator, and using the *ABCD* law, it requires that

$$q = \frac{Aq + B}{Cq + D} \qquad (4)$$

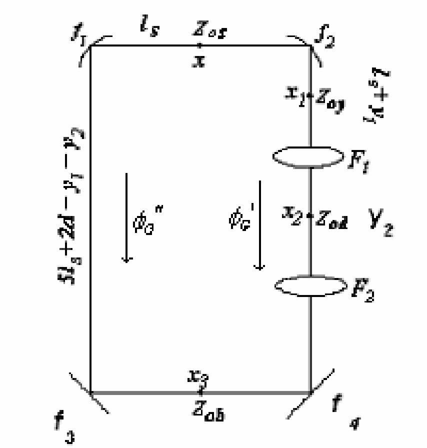
where *A*, *B*, *C*, *D* are the "ray' matrix elements for one complete round trip, starting and ending at the chosen reference plane, q denotes the complex beam parameter at this plane.

With the Gouy phase term ϕ_G, the condition for the frequency of oscillation in a ring cavity with perimeter of length L is obtained as

$$2n\pi = kL + \phi_G \qquad (5)$$

3. RESULTS AND DISCUSSION

A theoretical study is made to analyze the Gouy phase for the clockwise and anti clockwise beams traveling in a rotating ring laser. Despite of change in the optical path length, due to rotation, one can fix the length of the cavity in the case of no rotation. Without changing the length of the cavity, the distance between the optical elements is varied to develop a variable π phase shift. The arrangement studied for varying the Gouy shift is shown in the fig (1).



fig(1)

In this arrangement a ring cavity with four 100% reflecting mirrors is considered. Two mirrors f_1, f_2 are spherically concave mirrors and the other two f_3, f_4 are plane. To create four modes in the ring situation, a lens combination F_1, F_2 is inserted as shown. By varying the distance between these two lenses, the magnitude of the Gouy phase can be controlled within range of 0 to π. Note that there exist four Gaussian beam modes with half Rayleigh ranges $Z_{0s}, Z_{0y}, Z_{0d}, Z_{0b}$ in the considered configuration. And x, x_1, x_2, x_3 parameters are the corresponding positions of the beam waists with respect to the neighboring optical element.

In the absence of rotation, the frequencies of the oppositely traveling waves are (i.e., for the clock wise and for the anti clockwise) equal. As demonstrated by Sagnac, when the ring cavity is rotated, the degeneracy in optical path is removed. Thus the oppositely directed beams oscillate at different frequencies and the frequency difference is proportional to the rotation rate of the cavity. The conditions for the oscillation of the frequency for the clockwise ($L' = L - \Delta L$) and anti clockwise ($L' = L + \Delta L$) directions are the following:

$$\begin{aligned} 2n\pi &= k'L' + \phi_G{}' \\ 2n\pi &= k''L'' + \phi_G{}'' \end{aligned} \qquad (6)$$

$$\lambda' - \lambda'' = 2.\frac{\Delta L}{n} + \frac{L}{2n^2\pi}\left(\phi_G{}' - \phi_G{}''\right) + \frac{\Delta L}{2n^2\pi}\left(\phi_G{}' + \phi_G{}''\right) \qquad (7)$$

The above equation shows the beat frequency as a function of the differential Gouy phase. The differential Guoy phase $\left(\phi_G{}' - \phi_G{}''\right)$ is varying with the ΔL as shown in the fig (2).

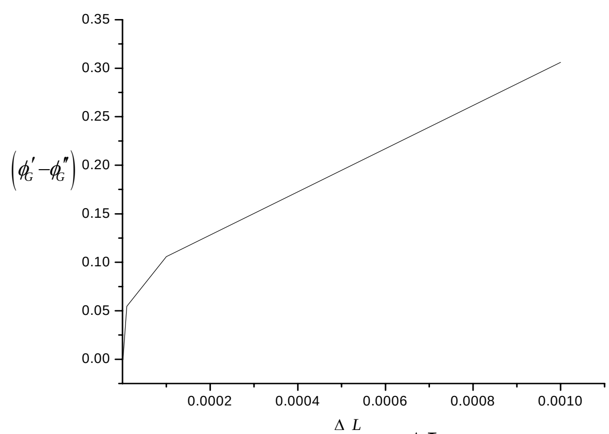


Fig (2): The differential Gouy phase is plotted by varying the values of ΔL . System parameters chosen are*: ls = 5cm, F= 15.106cm,* $f_1 = f_2$ *= 100cm and d =45cm.*

It has been observed that the differential Gouy phase term is giving significant change to the beat frequency equation.(2). The same configuration can be used to investigate the differential Gouy phase effect and help make the RLG more accurate.

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