Modified Allan Variance for Characterization of Ring Laser Gyro

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ABSTRACT

Heretofore, the "Allan Variance" $\sigma_y^2(t)$ has become the de facto standard for characterization of Ring Laser Gyro errors. The Allan variance has been demonstrated as a very useful statistical tool for characterizing various random processes with the exception that it can not distinguish between white PM and flicker PM noises. Moreover the Allan variance process has got limited frequency domain resolution capability. To overcome these shortcomings Modified Allan Variance has been developed and successfully applied to characterize high stability oscillators. In this paper Modified Allan variance has been used to characterize Ring Laser Gyro errors. By this variance method the White PM noise and Flicker PM noise can be evaluated separately. The confidence of estimation for estimated parameters is also improved. In this paper the analysis has been carried out on actual sensor data and the parameters have been evaluated.

Keywords: Error modeling, Ring Laser Gyro, Power law spectrum, Flicker noise, White noise

1. INTRODUCTION

Ring Laser Gyro contains a high precision optical resonator whose errors can be represented in terms of power law spectrum as it is very well standardized process for electronics oscillators' characterization. The random fluctuations in a high precision oscillaotor are very often characterized by the following power law.

$Sy(f)=h_\alpha f^{\alpha}$

where y is the normalized frequency deviation, f is the Fourier frequency, $h\alpha$ is the intensity of particular noise process and α is constant over some range of f. The typical values of σ lies from -2 to +2. The normal Allan variance has been defined as

$$\sigma_y^2(t) = \frac{1}{2\tau^2} < (x_{i+2} - 2x_{i+1} + x_i)^2 >$$

2. NOISE SOURCES IN RLG

RLG sensor error dynamics can be modelled by the discrete-time stochastic model presented in fig 1. In this model there are contributions from angular random walk, rate random walk, flicker noise, ramp noise and high frequency quantization noise. Makov noises and sinusoidal noises are considered to be negligible for our sensor and hence not considered for this study. This model can be represented in state-space form also which will be mathematically handled in simpler way.

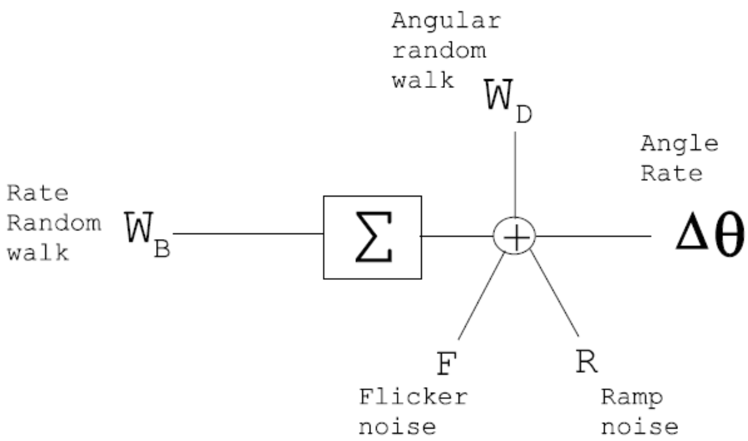

Fig 1. Stochaistic Model for RLG

2. NOISE ESTIMATION USING ALLAN VARIANCE METHOD

The data is collected for approximately 120 hrs and the algorithm is executed in the data. Applying this "Allan Variance" process to Ring Laser Gyro characterization gives the figure 2 and parameters. Here it can be observed that for α = +1 and +2 the dependency in *t is* nominally the same. In maximum cases white PM and flicker PM superimpose and it is difficult to differentiate between the noises.

The estimated parameters bias drift, quantization noise and angle random walk parameters match the theoretical parameters where as the long drift co-efficient and rate random walk are not accurate at all.

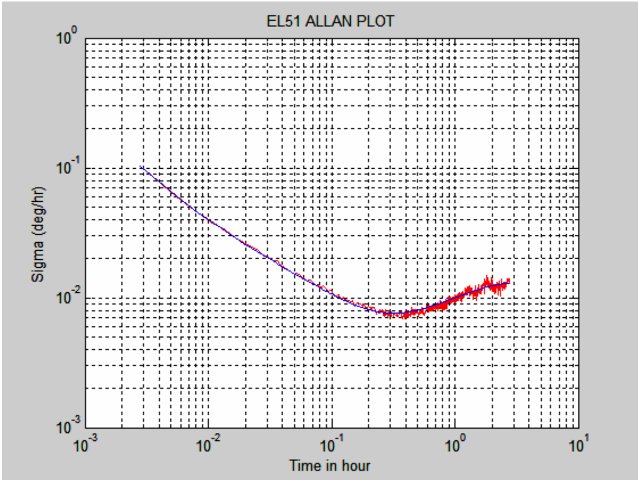

Fig 2. Allan Variance Estimation Plot

4. DEVELOPMENT OF MODIFIED ALLAN VARIANCE

The $\sigma_y^2(t)$ can be rewritten in terms of a generalized auto covariance function of x(t):

$$\sigma_y^2(t) = \frac{1}{2\tau^2}[4Ux(\tau) - Ux(2\tau)]$$

Where

Ux(τ)=2[Rx(0)-Rx(τ)], Rx(τ) being the auto-covariance of x(τ).

Using the Fourier transform of generalized function, one may determine the coefficients relating the power spectral density to σy2(τ).

Using the software bandwidth adapting technique the equation can be re-written as

$$Mod\ \sigma_y^2(\tau) = \frac{1}{2\tau^2}\sum_{i=1}^{n}(x_{i+2n} - 2x_{i+n} + x_i)^2$$

From this equation it can be seen that it is in general a second difference of three samples with each of the three samples being an average of n of the x_i's.

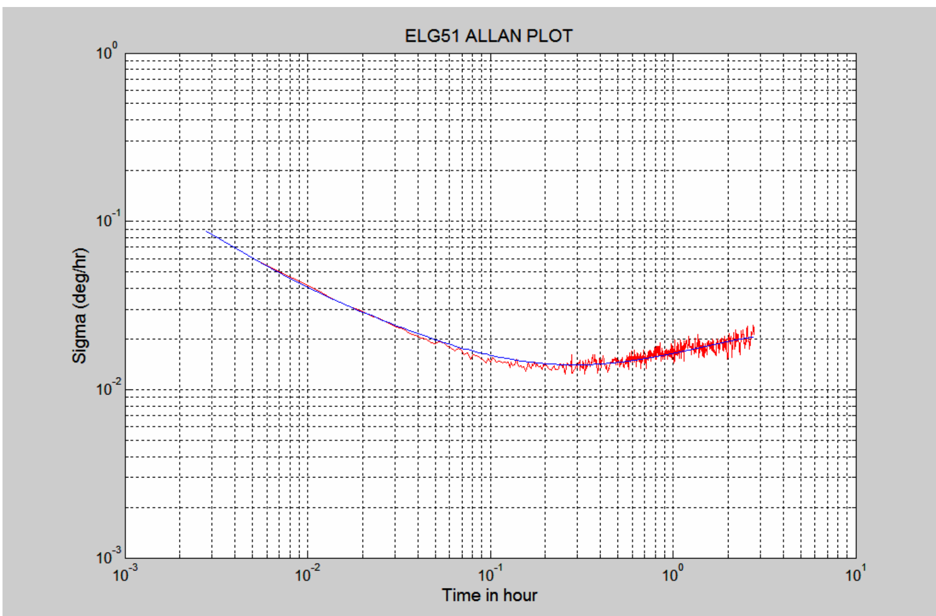

Fig 3: Modified Allan Plot

This analysis is carried out on the same set of data and figure 3 shows the result. Here the sinusoidal oscillations on gyro output can be easily seen which represent the flicker FM noise and white FM noise.

5. CONFIDENCE ESTIMATION OF MODIFIED ALLAN VARIANCE

Confidence estimation is defined in terms of equivalent degrees of freedom (edf).
Let N= no of time samples
m= sample interval period
M=N-3m+1
Q=M/m

Then

$$edf = \frac{a_0 q}{1 - {a_1}/{q}}$$

For our test data we have
$N=10^4$
m=100
M=9701
Q=M/m=97.01;

Taking these values the confidence factor for Random walk FM error for our sensor is calculated to be 85%. Earlier for normal Allan variance technique the confidence factor was only 78%.

6. CONCLUSION

The Modified Allan Variance technique has been applied to Ring Laser Gyro data analysis and it has been shown that the different types of error can be more accurately categorized and estimated with better confidence factor. Moreover this analysis can be used during calibration of sensor to get more accurate calibration parameters which will help improve the in-run performance of the sensor.

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REFERENCES

[1] W. W. Chow et al. "The Ring Laser Gyro", Review of Modern Physics, Vol 57, No.1, Jan 1985

[2] Anthony Lawrence, "*Modern Inertial Tecnology Navigation, Guidance and Control*" Second Edition

[2] Y.V. Bakin, G.N. Ziouzev and M.B. Lioudomirski "*Optical Gyros and their Application*", (Laser Gyro with Total Reflection Prisms, RTO AGARDograph 339), 1999

[3] David W. Allam et al. "A Modified Allan Variance with Increased Oscillator Characterisation Ability", 1981

[4] James A. Barnes et al. "Proc IEEE Transactions on Instrumentation & Measurement", IM-20, 1971

[5] Bernier, L.G. "**Theoretical Analysis of the Modified Allan Variance "**

[6] Das K. C. et al. "Application of Allan Variance to Ring Laser Gyro for Noise characterization and a signal processing technique for minimization of errors", Proceeding s of International conference on optics and Optoelectronics, 2005